# Light nuclei as quantum Skyrme solitons

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## Abstract

The Skyrme model (Skyrme 1961) has been extended to describe classical solitons with baryon number $B>1$ by rational map approximation (Manton and Sutcliffe 2004). The canonical quantization procedure (Acus et al. 1998) applied to rotational degrees of freedom of $^4$He and $^{16}$O nuclei leads to anomalous breaking of the chiral symmetry and exponential falloff of the energy density of the soliton at large distance, without explicit symmetry breaking terms included. Utilization of rational map ansatz yields a possibility of factorization of $S^3$ baryon charge into $S^1$ and $S^2$ parts, the phenomenology of the model being strongly affected by the chosen factorization. An interesting family of topological integrals on rational functions is discovered in the process of derivation of quantum equations of motion. The integrals is likely to indicate yet unknown discrete symmetries of the model. Calculated phenomenological radius, mass and electric formfactors show surprising Skyrme model selfconsistence between description of nucleon, $^4$He and $^{16}$O nuclei as well as reasonable correspondence with phenomenological data. The model is formulated for SU(2) representations of arbitrary dimension. The representation plays an interesting role additional model parameter. Quantization of classical states with predefined symmetry using different chiral left and right transformation matrices, like in (Acus et al. 2004), should be applicable to a wide class of nuclei with distinct spin and isospin values.

## Introduction

The chiral topological soliton model developed by Skyrme (Skyrme 1961) represents a dynamical realization of the large $N$ limit of QCD of baryons and nuclei (Manton and Sutcliffe 2004). The model describes baryons and nuclei as spatially extended topologically stable solitons of the chiral meson field. The soliton solutions of the equation of motion are characterized by the winding number or topological charge of the mapping $S^3 \to S^3$, which is interpreted as the baryon number $B$.

Numerical study has shown that the shape of the ground state field configuration for nuclei with $B>1$ has an intriguing geometrical structure (Battye and Sutcliffe 1997). For $B=2$ the ground state solution is toroidal and for $B=4$ the structure it is octahedral (see Figure 1). Higher baryon number solutions are associated with more complicated symmetric polyhedral shapes.

The rational map (RM) ansatz proposed for the SU(2) Skyrme model in (Houghton et al. 1998) provides a remarkably accurate analytic approximation to the ground state solution of the model. This ansatz preserves the essential symmetries of the numerical solutions of the exact Skyrme model equations. The identification of the topological number with baryon number also leads to solitonic fullerene structures in light atomic nuclei (Battye and Sutcliffe 2001). The rational map ansatz for the SU(2) skyrmion for $B=2$,which represents the deuteron, has been canonically quantized in Ref. (Acus et al. 2004) for representations of arbitrary dimension of the Skyrme model Lagrangian. The canonically quantized deuteron solutions and their physical characteristics depend on the dimension of the representation in contrast to the semiclassically quantized solution.

The matter density of the canonically quantized skyrmion soliton falls off exponentially at long range in contrast to the power law falloff of the classical soliton without a pion mass term (Acus et al. 2004, 1998). In the case of the $B=1$ skyrmion the inverse of the length scale of this exponential falloff corresponds to the pion mass, which arises because of the anomalous breaking of chiral symmetry by the canonical quantization procedure (Acus et al. 1998). In the case of the $\alpha$−particle it should correspond to $2\sqrt{mE_0}$, where $M$ is the nucleon mass and $E_0$ is the binding energy (Friar et al. 1985). Numerical calculation shows that the RM approximation leads to exponential falloff at a somewhat smaller rate than this. Below the static observables and the charge form factor of $^4$He are calculated from the the quantum solution of the $B=4$ skyrmion obtained with the rational map in SU(2) representations of arbitrary dimension. The calculated charge form factor has the same two-node structure as the experimental form factor, but the two zeros appear at smaller values of momentum transfer than in the empirical form factor. This shows that the ground state solution of the Skyrme model has an unrealistically compact structure, as expected.

## The Skyrme model

The Skyrme model is defined by the chirally symmetric Lagrangian

$$\mathcal{L} = -\frac{f_\pi^2}{4}\mathrm{Tr}\{\mathbf{R}_\mu \mathbf{R}^\mu\} + \frac{1}{32e^2}\mathrm{Tr}\{[\mathbf{R}_\mu,\mathbf{R}_\nu]^2\}, \tag{1}$$

Here the "right" current is defined as $R_\mu = (\partial_\mu U)U^\dagger$, and $f_\pi$ (the pion decay constant) and $e$ are parameters. $U(\mathbf{x},t)$ may be expressed as a direct sum of Wigner's $D$ matrices for irreducible representations $U(\mathbf{x},t) = \sum_j \oplus D^j(\alpha(\mathbf{x},t))$, which, in turn, are functions of three unconstrained Euler angles $\alpha = (\alpha^1,\alpha^2,\alpha^3)$. The rational map ansatz (Houghton et al. 1998) is an approximation to the ground state solution of the Skyrme model with baryon number $B>1$ takes the following form:

$$U_R(\mathbf{r}) = \exp(2\mathrm{i}\hat n^a \hat J_{(a)} F(r)) = D^j(\varkappa). \tag{2}$$

Here $\hat J_{(a)}$ are SU(2) generators in a given representation. The unit vector $\hat{\mathbf{n}}$ may be defined in terms of a rational complex function $R(z)$ as:

$$\hat{\mathbf{n}}_R = \frac{1}{1+|R|^2}\{2\Re(R), 2\Im(R), 1-|R|^2\}. \tag{3}$$

For baryon number $B=4$ the function:

$$R(z) = \frac{z^4+2\sqrt{3}\mathrm{i}z^2+1}{z^4-2\sqrt{3}\mathrm{i}z^2+1}, \tag{4}$$

has been found to be a suitable choice (Houghton et al. 1998). Here $z = \tan(\theta/2)\mathrm{e}^{\mathrm{i}\varphi}$ is a complex coordinate that is parametrized by azimuthal $\theta$ and polar $\varphi$ angles. The circular components of the unit vector $\hat{\mathbf{n}}_R$ are:

$$\begin{aligned}
\hat n_{+1} &= -\frac{1}{\sqrt{2}} + \frac{\sqrt{3}\sin^2\theta\left(\sqrt{3}\sin^2\theta - \mathrm{i}(1+\cos^2\theta)\cos 2\varphi\right)}{2\sqrt{2}\left(1-\sin^2\theta+\sin^4\theta(1-\sin^2\varphi+\sin^4\varphi)\right)},\\
\hat n_0 &= \frac{\sqrt{3}\sin^2\theta\cos\theta\sin 2\varphi}{1-\sin^2\theta+\sin^4\theta(1-\sin^2\varphi+\sin^4\varphi)},\\
\hat n_{-1} &= \frac{1}{\sqrt{2}} + \frac{\sqrt{3}\sin^2\theta\left(-\sqrt{3}\sin^2\theta - \mathrm{i}(1+\cos^2\theta)\cos 2\varphi\right)}{2\sqrt{2}\left(1-\sin^2\theta+\sin^4\theta(1-\sin^2\varphi+\sin^4\varphi)\right)}.
\end{aligned} \tag{5}$$

The rational map (4) has cubic symmetry. The orientation is fixed below so that the z-direction is that of the third component of the angular momentum. Differentiation of $\hat{\mathbf{n}}$ yields the relation

$$(-1)^s(\nabla_{-s} r\,\hat n_m)(\nabla_s r\,\hat n_{m'}) = \hat n_m \hat n_{m'} + I\left((-1)^m\delta_{-m,m'} - \hat n_m \hat n_{m'}\right), \tag{6}$$

which proves to be useful in the explicit calculation of Lagrangian density (1). Here $\nabla_s$ are the circular components of the nabla operator. The symbol $I$ here denotes the function:

$$I = \left(\frac{1+|z|^2}{1+|R|^2}\left|\frac{\mathrm{d}R}{\mathrm{d}z}\right|\right)^2, \tag{7}$$

the explicit form of which is:

$$I = \frac{12\sin^2\theta\,(1-\sin^2\theta+\sin^4\theta\,\sin^2\varphi\,\cos^2\varphi)}{\left(1-\sin^2\theta+\sin^4\theta\,(1-\sin^2\varphi+\sin^4\varphi)\right)^2}. \tag{8}$$

Integrals of powers of $I$ over $\theta$ and $\phi$ can be regarded as Morse functions (Houghton et al. 1998). The baryonic charge density takes the following form in the irrep $j$ :

$$\mathcal{B}(r,\theta,\varphi) = \varepsilon^{0k\ell m}\,\mathrm{Tr}\,R_k R_\ell R_m = -8\,j(j+1)(2j+1)\,I\,\frac{F'(r)\sin^2 F}{r^2}. \tag{9}$$

Because of the presence of the $I$ function in this expression, there is no need to modify usual boundary conditions $F(0)=\pi$; $F(\infty)=0$ for the chiral angle. The baryon number therefore takes the standard expression:

$$B = \frac{1}{24N\pi^2}\int_0^\infty \mathrm{d}r \int_0^{2\pi}\mathrm{d}\varphi \int_0^\pi \mathrm{d}\theta\ \mathcal{B}\, r^2\sin\theta, \tag{10}$$

with the normalization factor $N = \frac{2}{3}j(j+1)(2j+1)$. The present choice of boundary conditions ensures that the integral of the $I$ function is proportional to the baryon number:

$$\int_0^{2\pi}\mathrm{d}\varphi \int_0^\pi \mathrm{d}\theta\ I\sin\theta = 4\pi B. \tag{11}$$

Substitution of the rational map ansatz (2) into the Lagrangian density (1) leads to the classical Skyrme model density:

$$\mathcal{L}_{\mathrm{cl}} = -N\left(f_\pi^2\left(\frac{F'^2}{2} + \frac{I\sin^2 F}{r^2}\right) + \frac{1}{e^2}\frac{I\sin^2 F}{r^2}\left(F'^2 + \frac{I\sin^2 F}{2r^2}\right)\right). \tag{12}$$

Note, that the symmetry of the Lagrangian density (12) in the $\theta,\varphi$ space is completely determined by the function $I$ and its (more symmetric) powers (see Figure 1). It is useful to introduce dimensionless coordinates $\tilde r = e f_\pi r$. Variation of Lagrangian then yields the following differential equation for chiral angle:

$$\begin{aligned}
&F''(\tilde r)\left(1+\frac{2B\sin^2 F(\tilde r)}{\tilde r^2}\right) + \frac{2F'(\tilde r)}{\tilde r} + \frac{F'^2(\tilde r)B\sin 2F(\tilde r)}{\tilde r^2}\\
&-\frac{B\sin 2F(\tilde r)}{\tilde r^2} - \frac{I_2\sin^2 F(\tilde r)\sin 2F(\tilde r)}{\tilde r^4} = 0.
\end{aligned} \tag{13}$$

Here we have used the abbreviation:

$$I_2 = \frac{1}{4\pi}\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi \mathrm{d}\theta\, I^2\sin\theta. \tag{14}$$

In the limit $\tilde r \to \infty$, the equation (13) reduces to simple asymptotic form

$$F''(\tilde r) + \frac{2F'(\tilde r)}{\tilde r} - \frac{2BF(\tilde r)}{\tilde r^2} = 0. \tag{15}$$

From this the asymptotic large distance solution, which satisfies physical boundary conditions, can easily be obtained as:

$$F(\tilde r) = C_1 \tilde r^{-\frac{1+\sqrt{1+8B}}{2}}. \tag{16}$$

Here $C_1$ a constant to be determined later by numerical solution procedure. Note that eqs. (12–16) are valid for all $B$, provided that the corresponding function $I$ is used.

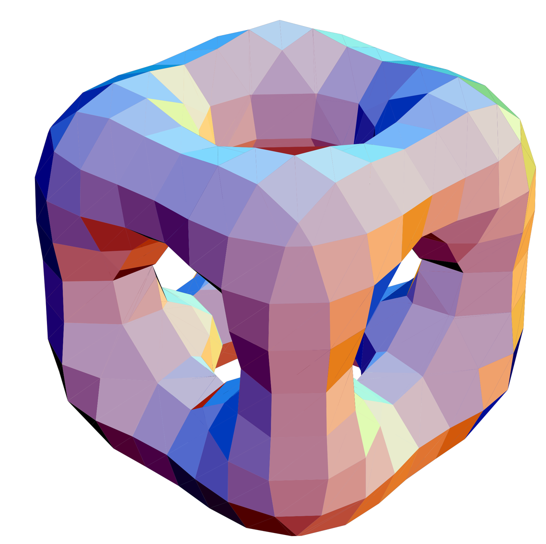

Figure 1. A typical surface of constant baryon density for $^4$He nuclei

## Quantization

The quantization of the Skyrme model in a general representation (Acus et al. 1998) can be carried out by means of collective rotational coordinates that separate the variables, which depend on the time and spatial coordinates (Adkins et al. 1983):

$$U(\mathbf{r},\mathbf{q}(t)) = A(\mathbf{q}(t))\,U_R(\mathbf{r})A^\dagger(\mathbf{q}(t)). \tag{17}$$

Here the three real Euler angles $\mathbf{q}(t) = (q^1(t), q^2(t), q^3(t))$ are quantum variables. These are sufficient for the $\alpha$ particle ground state, for which $S=T=0$. The canonical quantization with constraints procedure employed here was originally suggested by Dirac (Dirac 1964). In this formalism the Skyrme Lagrangian (1) is considered quantum mechanically *ab initio* in contrast to the conventional semiclassical quantization of the Skyrmion as a rigid body. In the SU(2) case canonical quantization implies that the three independent generalized coordinates $\mathbf{q}(t)$ and the corresponding velocities $\dot{\mathbf{q}}(t)$ satisfy the following commutation relations (Fujii et al. 1987):

$$[\dot q^a, q^b] = -\mathrm{i}f^{ab}(\mathbf{q}). \tag{18}$$

Here $f^{ab}(\mathbf{q})$ are functions of generalized coordinates $\mathbf{q}$ only, the explicit forms of which are determined self-consistently upon imposition of the quantization condition. The tensor $f^{ab}$ is symmetric with respect to interchange of the indices $a$ and $b$ by the relation $[q^a, q^b] = 0$. The commutation relation between a generalized velocity component $\dot q^a$ and an arbitrary function $G(\mathbf{q})$ is given by:

$$[\dot q_a, G(\mathbf{q})] = -i\sum_r f^{ar}(\mathbf{q})\frac{\partial}{\partial q^r}G(\mathbf{q}). \tag{19}$$

Here Weyl ordering of the operators has been employed:

$$\partial_0 G(\mathbf{q}) = \frac{1}{2}\{\dot q^\alpha, \frac{\partial}{\partial q^\alpha}G(\mathbf{q})\}. \tag{20}$$

The curly brackets denote an anticommutator. With this choice of operator ordering no further ordering ambiguity appears. To derive the Lagrangian the expression (17) is substituted into the Lagrangian density (1). Consider first the term that is quadratic in the generalized velocities. After integration over the spatial coordinates the Lagrangian takes the form:

$$L(\mathbf{q},\dot{\mathbf{q}},F) = \frac{1}{N}\int \mathrm{d}^3\mathbf{r}\,\mathcal{L}(\mathbf{r},\mathbf{q}(t),F(r)) = \frac{1}{2}\dot q^\alpha g_{\alpha\alpha'}\dot q^{\alpha'} + O(\dot q^0). \tag{21}$$

Here the momentum of inertia tensor is:

$$g_{\alpha\alpha'} = C'^{(b)}_\alpha(\mathbf{q})E_{(b)(b')}C'^{(b')}_{\alpha'}(\mathbf{q}). \tag{22}$$

Here $E_{(b)(b')}$ is defined as:

$$E_{(b)(b')} = -\frac{1}{2}(-1)^b a_b(F)\delta_{b,-b'} \quad \text{(no summation over } b\text{)}. \tag{23}$$

Here $a_1 = a_{-1}$. The soliton momenta of inertia are given as

$$\begin{aligned}
a_0(F) &= 4\pi\int_0^\infty r^2\sin^2 F\left((1-N_2)\left(f_\pi^2+\frac{1}{e^2}F'^2\right) + \frac{2}{3}\frac{B}{e^2}\frac{\sin^2 F}{r^2}\right)\mathrm{d}r\\
a_1(F) &= 2\pi\int_0^\infty r^2\sin^2 F\left((1+N_2)\left(f_\pi^2+\frac{1}{e^2}F'^2\right) + \frac{4}{3}\frac{B}{e^2}\frac{\sin^2 F}{r^2}\right)\mathrm{d}r
\end{aligned} \tag{24}$$

The symbol $N_k$ in this expression denotes the angular integrals:

$$N_k = \frac{1}{4\pi}\int_0^\pi \mathrm{d}\theta\int_0^{2\pi}\mathrm{d}\varphi\sin\theta\,\hat n_0^k. \tag{25}$$

For baryon number $B=1$ and $B=2$ the integrals may be evaluated in closed form to yield $N_2(\text{nucleon}) = \frac{1}{3}$; $N_4(\text{nucleon}) = \frac{1}{5}$ and $N_2(\text{deuteron}) = -1+\frac{\pi}{2}$, $N_4(\text{deuteron}) = -1/3+\frac{\pi}{4}$. For $B=4$ the numerical values of the corresponding integrals are $N_2 \approx 0.218$ and $N_4 \approx 0.118$. The other integrals, which enters calculation of the inertia tensor(23), may be evaluated analytically by the following expression:

$$\begin{aligned}
&\int\left(\frac{1+|z|^2}{1+|R|^2}\left|\frac{\mathrm{d}R}{\mathrm{d}z}\right|\right)^2\left(\frac{1-|R|^2}{1+|R|^2}\right)^m\frac{2\mathrm{i}\mathrm{d}z\mathrm{d}\bar z}{(1+|z|^2)^2} && m\geq 0\\
&= \int_0^\pi \mathrm{d}\theta\int_0^{2\pi}\mathrm{d}\varphi\sin\theta\, I\hat n_0^m = 2\pi B\frac{(-1)^m+1}{m+1}, && m \in \mathit{Rationals}
\end{aligned} \tag{26}$$

The validity of expression has been verified numerically for a number of randomly chosen rational maps with different baryon numbers $B$ to a very high degree of precision. There is good reason to conjecture that the integrals are topologically conserved quantities valid for all rational maps. Note that the relation (11) is a particular case ($m=0$) of eq. (26). Here the function $I$ plays an intriguing role as an "integrating" factor. The coefficients $C'^{(b)}_\alpha$ and their inverses $C'^{\alpha}_{(b)}$ are functions of the dynamical variables, which appear in the differentiation of the Wigner D matrices:

$$\frac{\partial}{\partial\alpha^k}D^j_{mn}(\alpha) = -\frac{1}{\sqrt{2}}C'^{(a)}_k(\alpha)D^j_{mm'}(\alpha)\langle jm'|J_{(a)}|jn\rangle. \tag{27}$$

The conventional quantum mechanical commutation relations $\left[p_\alpha, q^\beta\right] = -\mathrm{i}\delta_{\alpha\beta}$ for the momenta $p_\alpha = \frac{\partial L}{\partial\dot q^\alpha} = \frac{1}{2}\{\dot q^\beta, g_{\alpha\beta}\}$ then leads to the following expression for the tensor $f^{\alpha\beta}$ (18):

$$f^{\alpha\beta}(\mathbf{q}) = g^{-1}_{\alpha\beta}(\mathbf{q}). \tag{28}$$

It is convenient to introduce the following angular momentum operators on the hypersphere $S^3$ (the manifold of the SU(2) group):

$$\hat J'_{(a)} = -\frac{\mathrm{i}}{\sqrt{2}}\{p_\alpha, C'^{\alpha}_{(a)}\}. \tag{29}$$

It is readily verified that the operator $\hat J'_a$ is a $D^j(\mathbf{q})$ "right rotation" generator that has the well defined actions on the normalized state vectors with fixed spin and isospin $\ell$:

$$\left|\begin{matrix}\ell\\ m_s, m_t\end{matrix}\right\rangle = \frac{\sqrt{2\ell+1}}{4\pi}D^\ell_{m_s,m_t}(\mathbf{q})\,|0\rangle. \tag{30}$$

The explicit form of the function $f^{ab}(\mathbf{q})$, in turn, leads to to an explicit expression of the Skyrme model Lagrangian density (1) in the collective coordinate approach. To obtain explicit rezult requires lengthy manipulation using computer algebra system. At large distances the exact equation reduces to the asymptotic form:

$$\tilde r^2 F''(\tilde r) + 2\tilde r F'(\tilde r) - \left(2B + \tilde\mu^2\tilde r^2\right)F(\tilde r) = 0. \tag{31}$$

From this asymptotic equation it follows that the quantity $\tilde\mu$ describes the falloff rate of the chiral angle at large distances:

$$F(\tilde r) = C_1\mathrm{e}^{-\tilde\mu\tilde r}\left(\frac{\tilde\mu}{\tilde r} + \frac{B}{\tilde r^2}\right). \tag{32}$$

The related quantity $\mu = e f_\pi\tilde\mu$ describes the asymptotic falloff $\exp(-2\mu r)$ of the soliton mass density for the dimensional coordinate $r$. Integration and subsequent variation of Lagrangian density then leads to the integro-differential equation, which can be solved numerically.

## Numerical Rezults

The RM ansatz represents an approximation, which gives energies that fall above the numerically computed ground state energy by only a few percent (Battye and Sutcliffe 2001). Calculation of the static properties and the charge form factor of $^4$He from the RM with $B=4$ should therefore be expected to give a good approximation to those for the exact ground state solution. In the present numerical calculation the parameters $f_\pi$ end $e$ in the Skyrme model Lagrangian are determined so as to reproduce the calculated static observables of the nucleons in the different representations $j$ considered in Ref. (Acus et al. 1998). In that work the parameters were determined by the isoscalar radius (0.72 fm) and mass (939 MeV) of the nucleon. The same parameter values for $f_\pi$ and $e$ in the Skyrme model Lagrangian were employed here for the solution of the $B=4$ soliton, which describes the $^4$He nucleus in different representations. Fig. 2 shows the chiral angle profile functions for different baryon numbers 1, 2, 4 and 16.

Table 1. The predicted static $^4$He nuclei observables (Acus et al. 1998) in different representations with fixed empirical values for the nucleon isoscalar radius 0.72 fm and nucleon mass $m_N = 939$ MeV. The momenta of inertia, $\tilde a_i$, are in units of $1/(e^3 f_\pi)$.

| $j$ | 1/2 | 1 | 3/2 | $1\oplus\frac{1}{2}\oplus\frac{1}{2}$ | Exp. |
|---|---|---|---|---|---|
| $f_\pi$ | 59.8 | 58.5 | 57.7 | 58.8 | 93 MeV |
| $e$ | 4.46 | 4.15 | 3.86 | 4.24 | |
| $m$ | 3585 | 3759 | 3975 | 3701 | 3728.55 MeV |
| $\mu$ | 33.1 | 45.2 | 50.4 | 41.8 | 229? MeV |
| $\langle r_E^2\rangle^{1/2}$ | 1.39 | 1.52 | 1.65 | 1.49 | 1.676 fm |
| $E_0$ | −171 | +3 | +219 | −55 | −28.11 MeV |
| $\tilde a_0$ | 157.1 | 154.6 | 152.9 | 155.2 | |
| $\tilde a_1$ | 130.1 | 128.1 | 126.8 | 128.6 | |

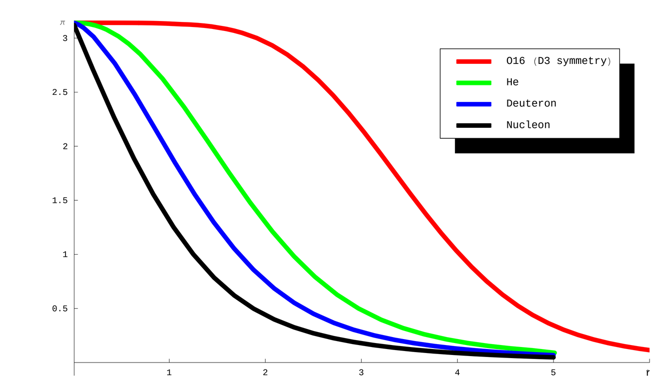


Figure 2. $^{16}$O, $^4$He, deuteron and nucleon (black) chiral angle profile functions in SU(2) representation $\frac{1}{2}$.

It is notable that the exponential falloff rate of the chiral angle becomes slower and smoother with increasing baryon number. The calculated values of the static observables of $^4$He are listed in Table 1.

The best agreement between the calculated and the empirical values for the charge radius $\langle r_E^2\rangle^{1/2}$ and the corresponding binding energy $E_0$ values is found for the reducible representation $1\oplus\frac{1}{2}\oplus\frac{1}{2}$ as in the case of the nucleon Acus et al. (1998). For the higher irreps no binding is found at all with these parameter values. While the finite pion mass is conventionally introduced by adding an explicitly chiral symmetry breaking pion mass term to the Lagrangian density of the model Adkins et al. (1983), the canonical quantization procedure by itself gives rise to a finite pion mass. This realizes Skyrme's original conjecture that "This (chiral) symmetry is, however, destroyed by the boundary condition ($U(\infty)=1$), and we believe that the mass (of pion) may arise as a self consistent quantal effect" (Skyrme 1962). The "quantal effect" (the exponential falloff rate of the mass density of $^4$He, $\mathrm{e}^{-2\mu r}$) which we find in (32) is, however, much smaller than the value that is obtained for a 4-nucleon system with the empirical binding energy: $\mu = \sqrt{mE_0}$, where $m$ denotes nucleon mass. The reason for this is that the rational map ansatz gives an approximation to the ground state solution, which does not contain the vibrational modes. This conclusion is also supported by comparison to the semi-classical approximation to the $B=4$ skyrmion given in ref. (Walhout 1992), which did take into account the vibrational modes, and obtained both a smaller binding energy (79 MeV) and concomitantly a larger radius (1.50 fm). Alternatively it may be viewed as natural consequence of the implied large $N$ limit of the model, in which there is no kinetic energy contribution from the constituent nucleons. The nonrelativistic charge form factors which are calculated from fixed empirical values of nucleon (Acus et al. 1998) have the same qualitative features as the empirical form factor values taken from Refs (Frosch et al. 1967; Arnold et al. 1967), with two nodes. The best agreement with experimental data is found for the fundamental representation $j=\frac{1}{2}$.

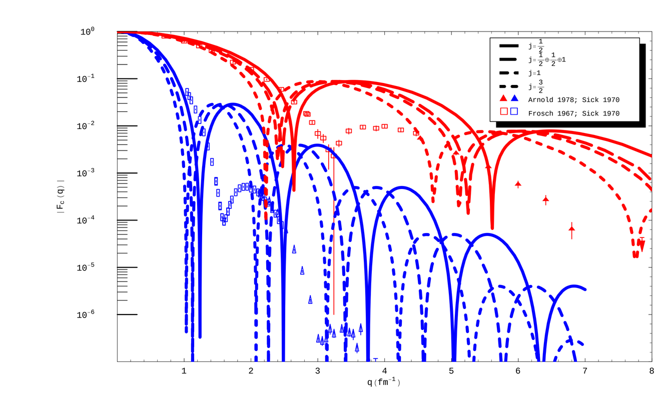


Figure 3. Comparison of $^4$He and $^{16}$O (D3 symmetry (Battye and Sutcliffe 2002)) electric form factors in different representations of SU(2) with experimental data (Frosch et al. 1967; Arnold et al. 1967; Sick and McCarthy 1970). The form factors are calculated with parameters that yield the experimental nucleon mass $m_N = 939$ MeV and radius $r = 0.72$ fm (Acus et al. 1998)

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